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De Sitter space in supergravity and M theory

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Abstract: Two ways in which de Sitter space can arise in supergravity theories are discussed. In the first, it arises as a solution of a conventional supergravity, in which case it necessarily has no Killing spinors. For example, de Sitter space can arise as a solution of $N = 8$ gauged supergravities in four or five dimensions. These lift to solutions of 11-dimensional supergravity or $D = 10$ IIB supergravity which are warped products of de Sitter space and non-compact spaces of negative curvature. In the second way, de Sitter space can arise as a supersymmetric solution of an unconventional supergravity theory, which typically has some kinetic terms with the "wrong" sign; such solutions are invariant under a de Sitter supergroup. These solutions lift to supersymmetric solutions of unconventional supergravities in $D = 10$ or $D = 11$, which nonetheless arise as field theory limits of theories that can be obtained from M-theory by timelike T-dualities and related dualities. Brane solutions interpolate between these solutions and flat space and lead to a holographic duality between theories in de Sitter vacua and euclidean conformal field theories. Previous results are reviewed and generalised, and discussion is included of Kaluza-Klein theory with non-compact internal spaces, brane and cosmological solutions, and holography on de Sitter spaces and product spaces.

Keywords: M-Theory, Superstring Vacua, Space-Time Symmetries.

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Contents

1. Introduction

There has recently been renewed interest in quantum gravity in de Sitter space and the issue of obtaining de Sitter space from string theory [1]–[13]. Here the ways in which de Sitter space can arise in supergravity theories will be examined, and the relevance of these for string theory will be discussed.

In [2, 14], it was argued that de Sitter space cannot arise from a conventional compactification of supergravity, string or M-theory. If d-dimensional de Sitter space is to emerge from string theory, it must therefore arise in some non-standard way. However it arises, the desired result would be to have a low-energy description in terms of a d-dimensional effective field theory in de Sitter space, and this theory should have local supersymmetries, which will typically all be spontaneously broken by the de Sitter solution. The constraints of local supersymmetry are very restrictive, but nonetheless there are theories with maximal local supersymmetry in $d = 4$ and $d = 5$ dimensions with de Sitter solutions of this type [15, 16]. These have a number of puzzling features, but the fact that there are so few examples consistent with maximal local supersymmetry suggests these are worth closer examination.

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Given such a d-dimensional theory, the next question is whether it can be obtained from string theory or M-theory, or from a solution of some D-dimensional supergravity ($D > d$). In most of the known cases, the D-dimensional origin of these de Sitter theories is as a solution which is a (possibly warped) product of d-dimensional de Sitter space and a $D - d$-dimensional "internal space" which is non compact. Thus the supergravity theory suggests that the way round the no-go theorem of [2, 14] is to have such a "non compactification". In general, solutions of this kind with non-compact extra dimensions will not have an effective description in terms of a d-dimensional theory and will be intrinsically D dimensional. However, it is possible that in some cases such solutions will lead to d-dimensional physics. Examples with large extra dimensions in which this is so are provided by [17]. In the present context, one approach is to impose suitable boundary conditions on the "internal" non-compact space so that there is a mass gap and a sensible d-dimensional spectrum with light fields and infinite towers of massive states [18, 19, 20]. A useful criterion is to seek solutions in which there is a consistent truncation to a d-dimensional supergravity theory, and the ones obtained by "lifting" a d-dimensional theory to D dimensions are usually of this kind.

It is perhaps worth recalling that when gauged supergravities were first found, it was thought that they were disappointingly unphysical as they had supersymmetric anti-de Sitter vacua which appeared to be unstable. For example, the gauged $N = 4$ supergravity in $D = 4$ of [21] has a complex scalar ϕ with potential

$$V = -\frac{1}{2}g^2(\cosh(a|\phi|) + 2)\,, \tag{1.1}$$

where g is the gauge coupling constant and a is a constant, which could be absorbed into the definition of ϕ. This has a maximum at $\phi = 0$ leading to a spacetime with negative cosmological constant

$$\Lambda = -\frac{3}{2}g^2 \tag{1.2}$$

and flat-space intuition suggested that a theory with such a scalar potential should be unstable. However, the anti-de Sitter vacuum was later shown to be completely stable [22] and more recently the physical relevance of anti-de Sitter vacua has become understood [23].

There are also gauged supergravities with de Sitter vacua, and these again appear unstable as the critical point is a maximum. For example, the gauged $N = 4$ supergravity in $D = 4$ of [24] again has a complex scalar ϕ with potential

$$V = -\frac{1}{2}g^2(\cosh(a|\phi|) - 2) \tag{1.3}$$

with a maximum at $\phi = 0$ leading to a spacetime with positive cosmological constant

$$\Lambda = \frac{1}{2}g^2 \tag{1.4}$$

and there are similar de Sitter vacua of gauged $N = 8$ supergravities in $D = 4$ [15] and $D = 5$ [16] (although none are known in $D = 7$). These were lifted to solutions of suprgravity in 10 or 11 dimensions in [19]. Such de Sitter vacua break all supersymmetries, as was to be expected, and while such an upside-down potential is less unstable in anti de Sitter space than in flat space, due to the Breitenlohner-Freedman mechanism, it is liable to be more unstable in de Sitter space. Nonetheless, it is remarkable that such a structure is forced by supersymmetry, and it would be worth investigating whether such theories could be of relevance. For example, there could be a long period of inflation in which the scalar field rolls slowly down the potential, after which the vacuum might decay to a different (and perhaps realistic) solution. These solutions will be discussed in section 2.

As well as these de Sitter solutions of conventional supergravities, de Sitter space also arises in solutions of certain "variant" supergravity theories [1, 25, 26]. There are de Sitter superalgebras [25, 27] — typically they are different real forms of the more familiar anti-de Sitter superalgebras — but they do not have unitary highest weight representations. There are field theory realisations of these as supergravity theories in which the lack of unitary representations is reflected in the fact that some of the fields have kinetic terms with the wrong sign. These have de Sitter solutions which are invariant under the full de Sitter supergroup, but the lack of unitarity makes these problematic field theories. However, these arise from string theories which are formally related to the conventional string theories by dualities (such as T-dualities in a compact time direction) suggesting that these theories might be worth re-examining in a string theory context. They come from 10- or 11-dimensional solutions which are a direct product of de Sitter space with a non-compact hyperbolic space, but in these cases the "internal space" can be compactified by identifying under the action of a discrete isometry group. (This is not the case for the de Sitter solutions of conventional supergravities described above, as the "internal space" does not have a suitable discrete isometry group that can be used for such an identification.) These variant supergravities do not satisfy the conditions assumed in the theorems of [2, 14]. One advantage of these theories is that the supersymmetry of the de Sitter background can give clues as to how to treat de Sitter space that might be more widely applicable. These will be discussed in sections 3 and 4.

In [1], it was argued that de Sitter space supergravity or string theory in D dimensions should have a holographic dual which is a $D-1$-dimensional euclidean conformal field theory, with the de Sitter group $\mathrm{SO}(D,1)$ acting as the conformal group in $D-1$-dimensional euclidean space. This arose from an argument similar to that used in [23] for the anti-de Sitter case. The variant supergravities have brane-like solutions that interpolate between flat space and the de Sitter solution, which arises as a near-horizon limit. However these branes are spacelike surfaces with effective worldvolume theories that are euclidean field theories, instead of the timelike surfaces of conventional branes which have lorentzian world volume field theories.

Following [23], it was argued in [1] that the string theory in the de Sitter space arising in the near-horizon limit should provide a dual description of the euclidean world-volume field theory, with the de Sitter suprgravity arising as a 't Hooft limit of the field theory. These variant supersymmetric theories appear to be non unitary, but the discussion of [11] suggest that non unitarity could be a typical feature of de Sitter holography. As in the anti-de Sitter case, such a duality arises in more general contexts and can be extended to cases with less supersymmetry. This will be discussed further in sections 7, 8 and 9.

The presence of a cosmological horizon in de Sitter space raises a number of issues as to how to quantise in a de Sitter vacuum, one being whether the degrees of freedom to be used should be those in a causally connected region [6] or ones in the whole space [10]. It is perhaps worth mentioning that tachyons, such as those that arise in some of the theories discussed above, can escape through a horizon and for such theories it would not be appropriate to limit considerations to degrees of freedom within the horizon.

Finally, the construction of [28] or [29] provides another class of variant supergravities with de Sitter solutions [29, 30]; these will not be discussed here.

2. De Sitter space from gauged supergravity

In $D = 4$ the Cremmer-Julia $N = 8$ supergravity theory [31], with scalars in the coset space $E_7/\,\mathrm{SU}(8)$, can be gauged by promoting a subgroup of the rigid E_7 symmetry to a local symmetry. The SO(8) gauging of [32] has a maximally supersymmetric anti-de Sitter vacuum and can be truncated to the gauged $N = 4$ theory with potential (1.1). In [15], gaugings with gauge group $\mathrm{CSO}(p,q,r)$ were obtained for all non-negative integers p,q,r with $p+q+r = 8$, where $\mathrm{CSO}(p,q,r)$ is the group contraction of $\mathrm{SO}(p{+}r,q)$ preserving a symmetric metric with p positive eigenvalues, q negative ones and r zero eigenvalues. Then $\mathrm{CSO}(p,q,0) = \mathrm{SO}(p,q)$ and $\mathrm{CSO}(p,q,1) = \mathrm{ISO}(p,q)$. In [33], it was argued that these are the only possible gauge groups. Note that despite the non-compact gauge groups, these are unitary theories, as the vector kinetic term is not the minimal term $k_{ab}F^a \cdot F^b$ contracted with the indefinite Cartan-Killing metric k_{ab}, but is $Q_{ab}(\phi)F^a \cdot F^b$ contracted with a positive definite scalar-dependent matrix $Q_{ab}(\phi)$. Of these theories, the ones with gauge groups $\mathrm{SO}(4,4)$ and $\mathrm{SO}(5,3)$ have de Sitter vacua arising at local maxima of the potentials, and the $\mathrm{SO}(4,4)$ theory includes the gauged $N = 4$ theory with potential (1.3) as a sub-sector [15]. In $D = 5$, the gauged $N = 8$ supergravities include those with gauge groups $\mathrm{SO}(p,6-p)$ [16, 34] and of these the $\mathrm{SO}(3,3)$ gauged theory has a de Sitter vacuum.

In each of these cases, the de Sitter vacua break all supersymmetries and break the gauge group $\mathrm{SO}(p,q)$ down to its maximal compact subgroup $\mathrm{SO}(p) \times \mathrm{SO}(q)$. The higher-dimensional origin of these theories was found in [19]. The gauged supergravities in $D = 4$ with gauge group $\mathrm{CSO}(p,q,r)$ arise from warped configurations of 11-dimensional supergravity with "internal" space of the form $\mathcal{H}^{p,q,r}$ where $\mathcal{H}^{p,q,r}$ is the hypersurface of $\mathbb{R}^{p+q+r}$ in which the real Cartesian coordinates z^A of $\mathbb{R}^{p+q+r}$ satisfy

$$\eta_{AB} z^A z^B = R^2 \,. \tag{2.1}$$

Here R is a constant "radius", and η_{AB} is a constant metric with p positive eigenvalues, q negative ones and r zero eigenvalues. The metric on the hypersurface $\mathcal{H}^{p,q,r}$ is the positive definite metric induced from the euclidean metric on $\mathbb{R}^{p+q+r}$. Thus $\mathcal{H}^{p,0,0}$ is a sphere S^{p-1}, $\mathcal{H}^{p,1,0}$ is the hyperboloid H^p, which is the coset space $\mathrm{SO}(p,1)/\,\mathrm{SO}(p)$, $\mathcal{H}^{p,q,0}$ is a hyperbolic space (a non-symmetric space with negative curvature) and $\mathcal{H}^{p,q,r} = \mathcal{H}^{p,q,0} \times \mathbb{R}^r$ is a generalised cylinder with cross-section $\mathcal{H}^{p,q,0}$. For the cylinders, the flat directions can be compactified to give $\mathcal{H}^{p,q,0} \times T^r$.

If $d\Omega^2_{p,q,r}$ is the metric on $\mathcal{H}^{p,q,r}$ induced from the euclidean metric on $\mathbb{R}^{p+q+r}$, then the solutions studied in [19] include those with $r = 0$ and warped product metrics in $D = d + p + q - 1$ dimensions of the form

$$ds^2 = f_1^2(y) dS_d^2(x) + f_2^2(y) d\Omega^2_{p,q,0}(y) \,, \tag{2.2}$$

where $dS_d^2(x)$ is the metric of a solution of the d-dimensional gauged supergravity theory with coordinates x, while y are intrinsic coordinates on $\mathcal{H}^{p,q,0}$. Here

$$\eta_{AB} = \mathrm{diag}\left(\mathbb{1}_p, -c^2\,\mathbb{1}_q\right) \tag{2.3}$$

for some constant c and the warp factors are given in terms of

$$L^2 = R^{-2} \left[\sum_{i=1}^{p} \left(z^i\right)^2 + c^4 \sum_{i=p+1}^{p+q} \left(z^i\right)^2 \right] \tag{2.4}$$

by

$$f_1 = L^a \,, \qquad f_2 = L^b \tag{2.5}$$

for some constants a, b. As in the Freund-Rubin ansatz, there is an antisymmetric tensor field strength that is proportional to the volume form of one of the two factors. The cases of interest here are the ones in which dS_d^2 is the metric on d-dimensional de Sitter space. In $d = 4$ the de Sitter solution of the $\mathrm{SO}(4,4)$ gauged theory arises from the solution of 11-dimensional supergravity of this form with

$$p = 4 \,, \qquad q = 4 \,, \qquad c^2 = 1 \,, \qquad a = \frac{2}{3} \,, \qquad b = \frac{-1}{3} \tag{2.6}$$

while the de Sitter solution of the $\mathrm{SO}(5,3)$ gauged theory arises from the solution of 11-dimensional supergravity of this form with

$$p = 5 \,, \qquad q = 3 \,, \qquad c^2 = 3 \,, \qquad a = \frac{2}{3} \,, \qquad b = \frac{-1}{3} \,. \tag{2.7}$$

The de Sitter solution of the SO(3,3) gauged theory in $d = 5$ arises from a similar solution of IIB supergravity with $d = 5$ and

$$p = 3\,, \qquad q = 3\,, \qquad c^2 = 1\,, \qquad a = \frac{1}{2}\,, \qquad b = \frac{-1}{2} \tag{2.8}$$

and with the self-dual 5-form field strength given in terms of the volume forms on dS_5 and the internal space.

These solutions can be found by an analytic continuation of the S^7 compactification of 11-dimensional supergravity and the S^5 compactification of IIB supergravity [19]. The sphere reductions have consistent truncations to the gauged supergravity sector, and it follows from the structure of the analytic continuation that the "non compactifications" from 11 dimensions on $\mathcal{H}^{4,4}$ or $\mathcal{H}^{5,3}$ or from IIB supergravity on $\mathcal{H}^{3,3}$ have consistent truncations to the corresponding gauged supergravities (similar arguments were used in [35]).

3. De Sitter supergravities

There is a class of variant supergravities which have maximally supersymmetric de Sitter vacua invariant under a de Sitter supergroup. By invariance, it is meant that the d-dimensional classical solution is invariant under the de Sitter isometry group SO$(d,1)$, the supersymmetry transformations generated by a (maximal) set of Killing spinors, and an R-symmetry group which is typically non compact. (The definition of corresponding conserved charges is problematic in de Sitter space.) The supergravity theories in general have some fields with kinetic terms with the wrong sign, as is needed for invariance under a linearly-realised non-compact R-symmetry. These theories are typically analytic continuations to different real forms of the gauged supergravities that give anti-de Sitter space. The first such theory, constructed in [25], was a variant form of $N = 2$ gauged supergravity. The usual gauged $D = 4, N = 2$ theory has gauge group U(1) with charged gravitini and a negative cosmological constant. In the variant form of [25], the sign of the vector field kinetic term is reversed and the theory has a positive cosmological constant.

Maximally supersymmetric generalisations of this with 32 supersymmetries were found in [1, 26]. The usual ungauged $N = 8$ supergravity in $d = 5$ has scalars in the coset $E_{6(+6)}/\,\mathrm{USp}(8)$ and in [1] a variant form (again in $4+1$ dimensions and with 32 local supersymmetries) was found with scalar coset structure $E_{6(+6)}/\,\mathrm{USp}(4,4)$; this will be referred to here as the $N = 8^*$ theory. As USp(4,4) is non compact, some of the scalar fields have kinetic terms of the wrong sign, as do many of the other matter fields. There are gauged versions of this $N = 8^*$ theory with gauge groups SO$(p, 6-p)$ and in particular that with gauge group SO(5,1) has a $d = 5$ de Sitter vacuum invariant under the de Sitter group SU$^*(4/4)$ with bosonic subgroup

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$\mathrm{SO}(5,1)\times\mathrm{SO}(5,1)=\mathrm{SU}^*(4)\times\mathrm{SU}^*(4)$ and with 32 fermionic generators, corresponding to the 32 Killing spinors [1]. The other $\mathrm{SO}(p,6-p)$ gauge groups can be obtained from the $\mathrm{SO}(5,1)$ gauging using the methods of [15].

Similarly, in $3+1$ dimensions, there is a variant $N=8^*$ supergravity with coset structure $E_{7(+7)}/\,\mathrm{SU}(4,4)$ [1] instead of the structure $E_{7(+7)}/\,\mathrm{SU}(8)$ of the Cremmer-Julia theory, and this has a gauging with gauge group $\mathrm{SO}(6,2)$ [26]. This has a de Sitter vacuum invariant under the super group $\mathrm{OSp}^*(4/8)$, with bosonic subgroup $\mathrm{SO}(4,1)\times\mathrm{SO}(6,2)$ and 32 supersymmetries. $N=8^*$ theories with gauge groups $\mathrm{CSO}(p,q,8-p-q)$ can be obtained from this by the methods of [15].

Such variant theories cannot arise from dimensional reduction of conventional supergravities, and their higher-dimensional origin must be from variant supergravities. The IIA^* and IIB^* supergravities in $9+1$ dimensions [1] are variant forms of the usual type-IIA and -IIB supergravities in which the kinetic terms of the Ramond-Ramond gauge fields all have the wrong sign. The bosonic actions are

$$S_{IIA^*}=\int d^{10}x\sqrt{-g}\left[e^{-2\Phi}\left(R+4(\partial\Phi)^2-H^2\right)+G_2^2+G_4^2\right]+\cdots \tag{3.1}$$

and

$$S_{IIB^*}=\int d^{10}x\sqrt{-g}\left[e^{-2\Phi}\left(R+4(\partial\Phi)^2-H^2\right)+G_1^2+G_3^2+G_5^2\right]+\cdots\,, \tag{3.2}$$

where the field equations from (3.2) supplemented by the constraint $G_5=*G_5$. The scalars in the IIB^* theory take values in the coset $\mathrm{SL}(2,\mathbb{R})/\,\mathrm{SO}(1,1)$, with the Ramond-Ramond scalar having a kinetic term with the reversed sign. This sign reversal leads to brane solutions that carry Ramond-Ramond charge being spacelike (E-branes) rather than timelike (D-branes).

Reducing either of the type-II^* theories on a (euclidean) 5-torus gives the ungauged $N=8^*$ supergravity in $4+1$ dimensions while reducing on a 6-torus gives the ungauged $N=8^*$ supergravity in $3+1$ dimensions. The 5-dimensional $N=8^*$ supergravity theory with gauge group $\mathrm{SO}(5,1)$ is obtained as a consistent truncation of the IIB^* theory in the solution $dS_5\times H^5$ [1], where H^d is the hyperbolic space

$$H^d=\frac{\mathrm{SO}(d,1)}{\mathrm{SO}(d)} \tag{3.3}$$

with isometry group $\mathrm{SO}(d,1)$, while d-dimensional de Sitter and anti-de Sitter spaces are the coset spaces

$$dS_d=\frac{\mathrm{SO}(d,1)}{\mathrm{SO}(d-1,1)}\,,\qquad AdS_d=\frac{\mathrm{SO}(d-1,2)}{\mathrm{SO}(d-1,1)} \tag{3.4}$$

with isometry groups $\mathrm{SO}(d,1)$ and $\mathrm{SO}(d-1,2)$, respectively.

The IIA^* theory cannot be obtained from any variant theory in 10+1 dimensions, but arises from compactifying a supergravity theory in $9+2$ dimensions on one of the timelike dimensions [26]. The de Sitter theory in 4 dimensions can be obtained from a solution of this M^* supergravity given by the product $dS_4 \times AdS_7$, with the gauge group SO(6, 2) arising as the isometry group of AdS_7. Regarding the AdS_7 as the internal space, there is a consistent truncation to the 4-dimensional variant gauged supergravity.

The dimensional reductions considered in this section are all analytic continuations of the sphere reductions of conventional supergravities, and so the consistency of the truncations of these to lower-dimensional supergravity theories implies the consistency of the reduction here also. Note that only certain analytic continuations can be consistent with supersymmetry, and for example 7-dimensional de Sitter space does not arise [26, 36, 37]; the possible analytic continuations of the $AdS_7 \times S^4$ solutions of 11-dimensional supergravity that do arise in this way are given in [36, 37].

4. String theory, time and duality

The solutions of 11-dimensional supergravity and IIB supergravity of section 2 are classical solutions of M-theory or IIB string theory, although the tachyonic scalar potential appears to signal an instability. The de Sitter supergravities and their origin in IIB^* supergravity or M^* theory are more problematic, as the field theory limits have terms in the supergravity lagrangian with the "wrong" sign, but the arguments of [1, 26] suggest that there is a formal link to the usual M-theory via dualities involving the time dimension, and these will now be reviewed, as they provide a formal link between the AdS supergravities and the de Sitter ones, and motivate the holography conjectures to be discussed in later sections.

Consider type-IIA or -IIB string theory in flat space-time but with time periodically identified, with $t \sim t + 2\pi R$. If the periodicity is extremely large then one might expect the physics to be similar to that in Minkowski space. Such a background is certainly a solution of the theory, but issues arise as to whether the quantum theory makes sense with periodic time. Quantum mechanics or quantum field theory with periodic time has a number of unusual features. With periodic boundary conditions in time, one can solve the Schrödinger or wave equations, giving quantised frequencies, and one can perform the functional integral and calculate quantum correlation functions, but the interpretation is problematic. There is not a conventional probabality interpretation in such circumstances, as the result of any "experiment" would be determined by what happened last time round, and the wave function would have already collapsed. However, the problems of quantum interpretation are similar to those that arise in addressing the quantum behaviour of the whole universe in a cosmological setting. Indeed, in a periodic cosmology in which the universe expands and then contracts and then repeats the cycle, so that time is periodic with a recurrence time given by the "lifetime" of the universe, the problems of interpretation become the same as those of quantum cosmology. There are many suggestions in the literature as to how such issues can be addressed, but whatever the resolution, there should be some description of the universe as a whole which is quantum in nature. String theory can presumably be formulated in a cosmological spacetime or in a spacetime with periodic time, and in both cases issues of interpretation arise.

If time is identified with radius R, energy is quantised in units of $1/R$ and it is sometimes suggested that this could be in conflict with the quantization of mass in string theory in units of the string mass m_s, unless the radius R were related to $1/m_s$. This is not so; the energy $E = p^0$ is quantised

$$E = \frac{2\pi n}{R} \tag{4.1}$$

for some integer n and the string physical state conditions for a state with momentum $p^\mu = (E, \mathbf{p})$ give

$$E^2 - \mathbf{p}^2 = m_s^2 N\,, \tag{4.2}$$

where N is an integer given in terms of the eigenvalues of the oscillator number operators. These two conditions would clearly be in conflict in general if the spatial momentum $\mathbf{p}$ vanished, but compactifying time breaks Lorentz invariance and one can no longer use a Lorentz transformation to go to the rest frame $\mathbf{p} = 0$. The two conditions are compatible with $\mathbf{p} \neq 0$, and for any given N, one can find an energy satisfying (4.1) and a momentum $\mathbf{p}$ such that (4.2) is satisfied, but $\mathbf{p}$ will be non zero if $m_s^2 R^2/4\pi^2$ is irrational.

If string theory exists with periodic time, its properties can be analysed by standard methods, and in particular one can perform a T-duality in the time direction. In the functional integral, time is Wick rotated to $\tau = -it$, and the Euclideanised functional integral with a periodic coordinate will exhibit T-duality. However, there are a number of different ways of continuing the Euclideanised theory back to lorentzian signature, depending on whether the periodic coordinate is continued back to a spacelike or a timelike coordinate, or is treated as the periodic euclidean time corresponding to a finite temperature. In the case at hand, a timelike circle of radius R is T-dual to a timelike circle of radius $R' = \alpha'/R$. Such a timelike T-duality takes the bosonic string to the bosonic string and the heterotic string to the heterotic string, but takes type-IIA or -IIB string theories to new theories, the IIB^* and IIA^* theories whose field theory limits are the IIB^* and IIA^* supergravity theories [1]. For spacelike T-duality, IIA string theory on a circle of radius R is T-dual to IIB string theory on a circle of radius $R' = \alpha'/R$. For timelike T-duality, IIA (IIB) string theory on a timelike circle of radius R is T-dual to IIB^* (IIA^*) string theory on a circle of radius $R' = \alpha'/R$ [1]. The "wrong" signs of the kinetic terms of the RR fields can be understood from matching the dimensional reductions of the supergravities on a timelike circle. For example, dimensional reduction of IIA supergravity on a timelike circle to 9 euclidean dimensions gives a RR scalar $C = C_0$ and 2-form $C_{ij} = C_{0ij}$

whose kinetic terms will be of the "wrong" sign, and if they are to come from the reduction of a RR scalar field and 2-form field in a dual IIB-like theory in $9+1$ dimensions, these fields must have kinetic terms of the "wrong" sign.

The IIB^* (IIA^*) string theory on a background with a timelike circle of radius R is then precisely the IIA (IIB) string theory on the dual timelike circle with radius $R' = \alpha'/R$, but written in terms of different variables. Either both theories exist, or neither do; if both exist, then the problems with one can be translated to the problems of the other via the duality. With periodic time, the "wrong" signs may not be as bad as they at first appear, as instability or loss of unitarity are not the problems that they would be in flat space. A theory which would be classically unstable in flat space, due for example to terms of the wrong sign in the action leading to an energy density which is not positive, would not be unstable with periodic time: any instability which started to grow would have to shrink again to satisfy the periodic time boundary conditions. Similarly, non unitarity is often associated with loss of "probability", but again the boundary conditions would result in any "probability" that is lost having to come back again. There is a similar escape clause for many of the problems usually associated with "wrong" signs, and it is conceivable that the theory could be consistent in a background with compact time. In the decompactification limit $R' \to \infty$, the theory in Minkowski space appears to be unstable, due to the non-positive kinetic energy.

This suggests that while the type-II^* theories in Minkowski space could be unstable (or worse), they could be better behaved in backgrounds with compact time. If so, it is also possible that other backgrounds of the type-II^* theories could be viable, and in particular the theory in de Sitter or comsological backgrounds may not be as bad as they at first appear. For example, as will be discussed in section 8, there are supersymmetric cosmological solutions with an expanding time-dependent geometry in which a RR gauge field grows with time, with a time dependence given in terms of $H(t)$, and it is not clear whether a solution in which a field grows as the universe expands should be viewed as an instability. (Some solutions of this type are obtained from an analytic continuation of brane solutions, in which a radial coordinate r is replaced by a time coordinate t.) The properties of the theory in such cosmological backgrounds deserve further study.

If the IIA^* theory in flat space needs a compact time, then M^* theory in flat space in $9+2$ dimensions would need both timelike dimensions to be periodic. Dimensionally reducing M^* theory from $9+2$ dimensions on a spacelike circle gives a IIA-like theory in a spacetime in $8+2$ dimensions, and further dualities generate type-II string theories in all spacetime signatures (s,t) with $s+t=10$, and leads to one further 11-dimensional theory, the M' theory in $6+5$ dimensions [26]. In each case, there is a supergravity theory, although the details are different in each case, as the properties of spinors are sensitive to the signature [26]. If type-II string theory exists in flat space with periodic time, then these other type-II string theories and the M^*, M' theories should also exist and are related to M-theory by chains of dualities. In this way, the de Sitter solutions of the IIB^* or M^* theories could be regarded as solutions of exotic phases of M-theory.

5. Non compactifications

De Sitter space can be obtained in supergravity theories in higher dimensions if the extra dimensions take the form of a non-compact hyberbolic space, and the no-go theorem of [2, 14] suggests that this will be generic, and that de Sitter spaces in M-theory will typically be accompanied by a non-compact internal space, motivating a reconsideration of such spaces as solutions. In the examples considered here, there is a consistent truncation to a lower-dimensional supergravity theory, arising from configurations in which fields in the internal space are in their ground states. The question then arises as to whether this can be extended to allow general configurations with non-trivial dependence on the internal space while still being able to extract sensible lower-dimensional physics.

In some cases, one can compactify a non-compact internal space. If the internal space is $\mathbb{R}^n$, one can identify points under a discrete subgroup of the translation group to obtain a torus T^n. Less trivially, for the hyperbolic space H^d, one can identify points under the action of a discrete subgroup of the isometry group $\mathrm{SO}(d,1)$ to obtain a compact space $\tilde{H}^d$ and consider a conventional compactification on $\tilde{H}^d$. Thus the solution $dS_5 \times H^5$ of the IIB^* theory can be replaced by the compactifying solution $dS_5 \times \tilde{H}^5$. With such a compact internal space, one can dimensionally reduce in the standard way. However, this will not work for the spaces $\mathcal{H}^{p,q,0}$ with isometry group $\mathrm{SO}(p) \times \mathrm{SO}(q)$ as they do not have a non-compact "translational" isometry group that can be used in such an identification.

For a non-compact internal space, a standard dimensional reduction of the action leads to a reduced theory with Newton's constant inversely proportional to the volume of the internal space, so that it would vanish for an infinite volume internal space. However, for the classical theory, it is sufficient that the dimensionally reduced field equations make sense and this is possible even for an infinite volume internal space. In many cases, boundary conditions can be imposed so as to obtain a discrete spectrum. Consider for example a theory on $M \times N$, where N is regarded as the "internal" space and consider a scalar field $\Phi(x, y)$ satisfying a wave equation

$$\Delta\Phi = -m^2\Phi\,, \tag{5.1}$$

where Δ is the Laplace operator on $M \times N$, x are the coordinates on M and y those on N, and the laplacian splits into laplacians acting on M and N, $\Delta = \Delta_M + \Delta_N$. It was shown in [20] that in many cases the spectrum of the Laplace operator Δ_N consists of a discrete spectrum of normalisable modes and a continuous spectrum of non-normalisable ones. If boundary conditions can be imposed on N that eliminate the non-normalisable modes, one is left with a discrete set of eigenfunctions $f_n(y)$ of the laplacian Δ_N satisfying

$$\Delta_N f_n = \lambda_n f_n \,. \tag{5.2}$$

In such cases, a Kaluza-Klein-type spectrum emerges. A similar analysis extends to higher spins and to warped products [20].

Consider next the interactions. Suppose there is set of scalar fields Φ^a on $M \times N$ labelled by a with field equations

$$\Delta\Phi^a = -M^a{}_b\Phi^b + c^a_{bc}\Phi^b\Phi^c + O\left(\Phi^3\right) \tag{5.3}$$

with mass matrix $M^a{}_b$ and coupling constants c^a_{bc}. Then the decomposition

$$\Phi^a(x,y) = \sum_n \phi^a_n(x) f_n \tag{5.4}$$

will lead to well-defined field equations on M provided the eigenfunctions satisfy a completeness relation

$$f_m(y) f_n(y) = \sum_p d^p_{mn} f_p(y) \tag{5.5}$$

for some finite constants d^p_{mn}. Then the resulting field equations on M are

$$\Delta_M \phi^a_n = M^a{}_b\phi^b_n - \lambda_n\phi^a_n + c^a_{bc} d^n_{pq}\phi^b_p\phi^c_q + O\left(\phi^3\right) . \tag{5.6}$$

This extends to other interactions and general spins, with the result that the field equations on $M \times N$ can be reduced to well-defined field equations on M, much as in a compactification, provided boundary conditions are imposed on the fields on N such that the wave operators on N all have discrete spectra and in addition the eigenfunctions satisfy appropriate completeness conditions.

6. Holography in product spaces

Supergravity or string theories in a space M can have a holographic description in terms of a theory on the boundary of M, ∂M. In [37], this was generalised to theories on a product space $M \times N$ in which both M and N had boundaries, and it was argued that two dual field theories can play a role here, one on ∂M and one on ∂N. The boundary of $M \times N$ has two components, $\partial M \times N$ and $\partial N \times M$, and in general there could be holographic dual theories on these two boundaries. Suppose N is compact, or that boundary conditions are applied so that there is a discrete spectrum and the theory on $M \times N$ can be regarded as a theory on M with an infinite tower of massive fields, as discussed in the last section. Then there could be a dual description as a theory on the boundary ∂M of M (and similarly with the roles of M and N interchanged).

For theories on a supersymmetric $AdS_d \times S^n$ background, there is a holographic description in terms of a $d-1$-dimensional conformal field theory (CFT), which can be thought of as being on the $d-1$-dimensional boundary of AdS_d [23], with the AdS isometry group $\mathrm{SO}(d-1,2)$ acting as the conformal group on the boundary. There is a correspondence between boundary values of fields in anti-de Sitter space and operators in the conformal field theory, and in particular the Kaluza-Klein modes representing fields with non-trivial dependence on the internal dimensions correspond to certain operators in the CFT.

The correspondence can be formulated in terms of the Wick rotated theory, in which the AdS_d space is analytically continued to the d-dimensional hyperbolic space $H^d = \mathrm{SO}(d,1)/\mathrm{SO}(d)$ and the boundary theory is continued to a Euclideanised conformal field theory on the boundary S^{d-1} of H^d [38]. As in [1], it will be convenient to refer to supersymmetric field theories formulated directly in euclidean space as "euclidean", and to the theories obtained by Wick rotating supersymmetric theories from lorentzian space as "Euclideanised'; the Euclideanised theories will usually not have a conventional supersymmetry. For example, Wick rotating lorentzian $N = 4, D = 4$ super Yang-Mills gives a Euclideanised theory in $4+0$ dimensions with $\mathrm{SO}(6)$ R-symmetry and no conventional supersymmetry. A euclidean super Yang-Mills theory with 16 supersymmetries in $4+0$ dimensions and $\mathrm{SO}(5,1)$ R-symmetry is obtained by reducing super Yang-Mills from $9+1$ dimensions on 5 space and one time dimension [39, 40].

A similar situation applies for other products of non-compact spaces, when each has a holographic dual. Consider the product $dS_n \times H^d$, with isometry group $\mathrm{SO}(n,1) \times \mathrm{SO}(d,1)$. Both de Sitter space and hyperbolic space are non compact, and it was argued in [1, 37] that there should be two holographic descriptions arising from different limits, one which is a $d-1$-dimensional euclidean field theory on the boundary of H^d and one which is an $n-1$-dimensional euclidean field theory associated with de Sitter space. The space $dS_n \times H^d$ can be Wick rotated to the Euclideanised solution $S^n \times H^d$, which is exactly the same as the Euclideanisation of $AdS_d \times S^n$. This Euclideanised theory has a holographic description as a theory on the boundary of H^d, which continues back to a holographic description of the $dS_n \times H^d$ theory on the boundary of H^d. Here $\mathrm{SO}(d,1)$ acts as the conformal group on the euclidean d-dimensional CFT while $\mathrm{SO}(n,1)$ arises as the R-symmetry group. As de Sitter space becomes a sphere on Euclideanisation, the Euclideanised theory does not help in formulating the de Sitter holography. In [1] it was argued that in certain circumstances, to be discussed in the following sections, the physics in $n+1$-dimensional de Sitter space has a holographic description as an n-dimensional euclidean conformal field theory with conformal group $\mathrm{SO}(n,1)$ and R-symmetry $\mathrm{SO}(d,1)$. Thus there are two holographic duals: one on the boundary of H^d with conformnal group $\mathrm{SO}(d,1)$ and one associated with the boundary of de Sitter space with conformal group $\mathrm{SO}(n,1)$.

For example, the $AdS_5 \times S^5$ solution of IIB string theory has a holographic representation as $D = 4, N = 4$ super Yang-Mills theory on the boundary of AdS_5, with super-AdS group SU$(2,2/4)$ realised as the superconformal group, with conformal group SO$(4,2)$ and R-symmetry group SO(6). Wick rotation takes this to a Euclideanised theory on $H^5 \times S^5$ with isometry group SO$(5,1)\times$SO(6), and the holographic dual is the Euclideanised super-Yang-Mills theory with R-symmetry SO(6), the SO$(3,1)$ Lorentz symmetry continued to SO(4) and the conformal group continued to SO$(5,1)$. Neither theory has conventional supersymmetry, due to the usual problems in continuing spinors, self-dual forms and supersymmetries to euclidean space. (Supersymmetric theories can be obtained after further sign changes, so that the R-symmetry becomes SO$(5,1)$, to give euclidean supersymmetric theories.)

The $dS_5 \times H^5$ solution of the IIB^* theory is invariant under the super-de Sitter group SU$^*(4/4)$, which contains the isometry group SO$(5,1) \times$ SO$(5,1)$. This has a holographic dual description [1] in terms of a euclidean superconformal Yang-Mills theory in four euclidean dimensions obtained by reducing super-Yang-Mills from $9+1$ dimensions on one time and five space dimensions [39, 40]. This has conformal group SO$(5,1)$ and R-symmetry group SO$(5,1)$. Here $n = d = 5$, but the euclidean conformal field theory can arise in two ways, one on the boundary of H^5 and one associated with the de Sitter space, as will be reviewed in section 9. The Euclideanised version of this theory is the same theory on $H^5 \times S^5$ as obtained from continuing the $AdS_5 \times S^5$ solution. The euclidean super-Yang-Mills theory has 5 scalars with kinetic terms of the "right" sign and one of the "wrong" one, so that the SO$(5,1)$ R-symmetry can be linearly realised on them, and the Euclideanisation involves multiplying the wrong-sign scalar by i to get a positive action and SO(6) R-symmetry (just as in the string theory path integral, one continues $X^0 \to iX^0$).

7. Euclidean branes

T-duality exchanges Dirichlet and Neumann string boundary conditions. A Dirichlet p-brane at $y^i = 0$ in flat spacetime with coordinates $X^M = (t, x^1, \ldots, x^p, y^1, \ldots, y^{9-p})$ corresponds to strings $X^M(\sigma, \tau)$ with Neumann boundary conditions on the $p+1$ longitudinal coordinates t, x^i and Dirichlet boundary conditions on the transverse coordinates y^i. A T-duality in a particular direction changes the boundary conditions in that direction from Dirichlet to Neumann or vice versa, so a T-duality in a longitudinal spatial direction x^p takes it to a $p-1$-brane with p longitudinal coordinates $t, x^1, \ldots, x^{p-1}$. If the time direction is taken to be compact, a T-duality in the time direction takes a type-II theory to a type-II^* theory and changes the boundary conditions on $t(\sigma, \tau)$ from Neumann to Dirichlet, giving a p-dimensional spacelike surface parameterised by the coordinates $x^1, \ldots, x^p$ located at $y^i = 0$ and at a fixed moment in time, $t = t_0$ for some constant t_0. This is the Ep-brane of the type-II^* theory [1].

The world-volume theory on a stack of N Dp-branes is the $p+1$-dimensional super Yang-Mills theory with gauge group U(N) and R-symmetry SO$(9-p)$ obtained by reducing $9+1$-dimensional super Yang-Mills on a $9-p$ torus. For a single D-brane, there are $9-p$ scalar fields which are collective coordinates representing the position of the brane; changing their expectation values changes the position of the brane in the euclidean transverse space. For N Ep-branes, the world-volume theory is the p-dimensional euclidean super Yang-Mills theory with gauge group U(N) and non-compact R-symmetry SO$(9-p,1)$ obtained by reducing $9+1$-dimensional super Yang-Mills on a lorentzian torus with $9-p$ spacelike circles and one timelike circle. Again the expectation values of the $10-p$ scalars correspond to the position in the transverse space, but here the transverse space is lorentzian. This is reflected in the fact that $9-p$ of the scalars have the conventional sign of kinetic term, representing the spatial position of the brane, and one has the wrong sign kinetic term and changing its expectation value changes the brane instant $t=t_0$. For $p=4$, this gives the conformally invariant super Yang-Mills theory in four euclidean dimensions.

8. Interpolating solutions and branes

For a wide class of theories, there are m-brane solutions in $D=n+m+2$ dimensions with metric of the form

$$ds^2 = H^{-\delta}\left(-dt^2+dx_1^2+\cdots+dx_m^2\right)+H^{\alpha}\left(dr^2+r^2d\Omega_N^2\right), \tag{8.1}$$

with

$$H = c+\frac{a^{n-1}}{r^{n-1}}, \tag{8.2}$$

where c, a, α, δ are constants, $d\Omega_N^2$ is the metric on some n-dimensional space N (N is an n-sphere S^n for the most symmetric solutions, and is an Einstein space in typical cases). There may be a scalar field with non-trivial r dependence of the form

$$e^{\phi} = H^{\gamma} \tag{8.3}$$

for some constant γ. Near $r=0$, which is a horizon if $\delta>0$, the constant term in H can be dropped and the metric becomes conformal to a metric $d\tilde{s}^2$ on the product of $m+2$-dimensional AdS space and N,

$$ds^2 = H^A d\tilde{s}^2\,, \qquad A=\alpha-\frac{2}{n-1} \tag{8.4}$$

and

$$d\tilde{s}^2 = \frac{r^{\beta}}{a^{\beta}}dx_{||}^2+\frac{a^2}{r^2}dr^2+a^2d\Omega_N^2\,, \tag{8.5}$$

where the longitudinal metric is $dx_{||}^2=-dt^2+dx_1^2+\cdots+dx_m^2$ and $\beta=2-(n-1)(\alpha+\delta)$. For some physical questions, it is more natural to work with the "dual frame" metric

$d\tilde{s}^2$ rather than ds^2 [41]. The change of variables $U = r^{\beta/2}$ brings the metric on the AdS space to the form

$$ds^2_{\text{AdS}} = \frac{U^2}{a^\beta} dx^2_{||} + \frac{4a^2}{\beta^2} \frac{1}{U^2} dU^2 \tag{8.6}$$

with dilaton

$$e^\phi \propto U^{-2(n-1)\gamma/\beta} . \tag{8.7}$$

The constants in (8.6) can be absorbed into rescalings of the coordinates. For the space (8.1), as r becomes large, H approaches the constant c and the space approaches flat space $\mathbb{R}^D$ if N is a round sphere S^n, or the product of $m+1$-dimensional Minkowski space and a cone over N otherwise.

This interpolating brane solution is the basis for arguments that the theory in the "near horizon geometry" (8.6) is holographically dual to an $m+1$-dimensional theory, which is the world-volume theory of the brane. For the D3-, M2- and M5-branes, $A = 0$ and the scalar is either absent or constant, and the dual theory is a superconformal field theory. The solution is invariant under the AdS group $\mathrm{SO}(m-1, 2)$ and the $\mathrm{SO}(n+1)$ isometry group of $N = S^n$, and these, together with the 32 supersymmetries, form a super-AdS group which is interpreted as the superconformal group of the dual conformal field theory. For other branes, with $N = S^n$ but $\gamma \neq 0$, the near horizon limit is again $AdS_{m+2} \times S^n$ in the dual frame, but the r-dependence of the dilaton breaks the AdS group down to a Poincaré group, and the dual theory is non conformal, with the dependence of the string coupling on the radial coordinate r or U corresponding to a scale dependence of the dual field theory coupling constant. The $\mathrm{SO}(n+1)$ remains as an R-symmetry of the dual theory. In the more general case in which N is not a sphere, the dual theory is the world-volume theory of a cone at a conical singularity, with R-symmetry given by the isometry group of N. Note that this formally extends to the cases in which N is non compact or has a metric of non-euclidean signature.

These solutions interpolating between AdS $\times$ N and flat space (or a cone) have analogues for solutions interpolating between a de Sitter solution dS $\times$ N and flat space or a generalised cone. A similar statement may also be true of the warped products of section 2, but here attention will be restricted to the direct product solutions of sections 3 and 4, for which the interpolating solutions and some generalisations have been given in [1, 26, 36]; these solutions generically preserve some of the supersymmetries.

Consider then a brane-like space in $D = n + m + 1$ dimensions with metric

$$ds^2 = H^{-\delta} \left(dx_1^2 + \cdots + dx_m^2 \right) + H^\alpha \left(-dt^2 + dr^2 + r^2 d\Omega_N^2 \right) , \tag{8.8}$$

where H is a function of r and t in general, c, α, δ are constants and $d\Omega_N^2$ is the metric on some $n-1$-dimensional space N. While the solution (8.1) is interpreted as representing an object extended in m space and one time dimension that is localised at $r \sim 0$ and is a timelike surface parameterised by $t, x_1, \ldots, x_m$, the solution (8.8) represents a "euclidean-brane" which is an m-dimensional spacelike surface parameterised by $x_1, \ldots, x_m$. It can be localised in the transverse space parameterised by t, r and the coordinates of N, with the form of the "localisation" depending on the choice of H, which is a harmonic function on the transverse space. As before there will in general be a scalar field of the form

$$e^{\phi} = H^{\gamma} \tag{8.9}$$

for some constant γ.

One class of solutions is that in which H is independent of t and given by

$$H = c + \frac{q}{r^{n-2}} \,. \tag{8.10}$$

Such branes arise from time-like T-dualities; for example, starting with a Dp-brane of type-II string theory with $p = m$ in a background with periodic time, then performing a T-duality in the time direction gives an E-brane solution of the form (8.8) [1]. The T-dual of an AdS solution in the compact time direction is a solution of this form with $c = 0$, so that the constant term is absent in the harmonic function.

Another class of solutions has H independent of r and given by a linear function of t,

$$H = mt + b \,. \tag{8.11}$$

This gives a "cosmological" solution (which can be viewed as an analogue of the domain-wall spacetimes which are of the form (8.1) and (8.3) with H having a linear dependence on one of the spatial coordinates). The theories of [1, 26] have many supersymmetric solutions of this form.

Next, there are solutions in which H depends on the proper time $\tau^2 = t^2 - r^2$

$$H = c + \frac{a^{n-1}}{\tau^{n-1}} \,, \tag{8.12}$$

where for regularity in the region $t^2 > r^2$ we take $c \geq 0, a^n \geq 0$. Again it is useful to define a conformally related dual frame metric $d\tilde{s}^2$ of the form (8.4). The space with metric $d\tilde{s}^2$ and coordinates restricted to the region $\tau^2 \geq 0$ is geodesically complete and non singular [1]. As τ^2 becomes large, H tends to a constant and the space with metric ds^2 or $d\tilde{s}^2$ tends to flat space if N is a round sphere, or to a product of $m+1$-dimensional Minkowski space and a cone over N otherwise. Near $\tau = 0$, the constant term in H can be neglected so that $H \sim (a/\tau)^{n-1}$. To study the behaviour near $\tau = 0$, it is useful to define the Rindler-type coordinates τ, ρ by

$$t = \tau \cosh \rho \,, \qquad r = \tau \sinh \rho \tag{8.13}$$

so that the metric $d\tilde{s}^2$ becomes

$$d\tilde{s}^2 = \frac{\tau^{\beta}}{a^{\beta}} dx_{||}^2 - \frac{a^2}{\tau^2} d\tau^2 + a^2 d\tilde{\Omega}_{\tilde{N}}^2 \,, \tag{8.14}$$

where $d\Omega^2_{\tilde N}$ is the metric on the n-dimensional space $\tilde N$ with metric

$$d\Omega^2_{\tilde N} = d\rho^2 + \sinh^2\rho\, d\Omega^2_N\,. \tag{8.15}$$

If N is a round $n-1$ sphere S^{n-1}, then $\tilde N$ is the hyperbolic space H^n, the coset space $\mathrm{SO}(n,1)/\mathrm{SO}(n)$, with "radius" 1. The longitudinal metric is now the euclidean metric $dx^2_{||} = dx_1^2 + \cdots + dx_m^2$. Then

$$ds^2 = \frac{\tau^\beta}{a^\beta} dx^2_{||} - \frac{a^2}{\tau^2} d\tau^2 \tag{8.16}$$

is the metric on $m+1$-dimensional de Sitter space, dS_{m+1}. The change of variables $T = (\beta/2a^{1+\beta/2})\tau^{\beta/2}$ brings the metric on the dS space to the form $(4/\beta^2)ds^2_{\mathrm{dS}}$ where

$$ds^2_{\mathrm{dS}} = \frac{T^2}{a^2} dx^2_{||} - \frac{a^2}{T^2} dT^2 \tag{8.17}$$

with dilaton

$$e^\phi \propto T^{-2(n-1)\gamma/\beta}\,. \tag{8.18}$$

In the limit $c=0$, the coordinates $T, x_{||}$ with $T>0$ cover only half of de Sitter space. There is a coordinate singularity at $T=0$, and the geometry can be continued through this to give the complete non-singular de Sitter solution, with $T=0$ the cosmological horizon. The transverse space has metric $-dt^2 + dr^2 + r^2 d\Omega^2_N$ and the region $t^2 > r^2$ consists of the interior of the past and future light cones of the origin. The interior of the light-cone splits into two regions, the past light-cone $t < r < 0$ and the future light-cone $0 < r < t$, and it is natural to define the proper time so that these are the two regions $T < 0$ and $T > 0$, and correspond to the two halves of the de Sitter space [1]. For the metric $d\tilde s^2$ with $t^2 > r^2$, the region near $t^2 = r^2$ or $\tau = 0$ is described by a non-singular $\mathrm{dS}\times\tilde N$ geometry, and $\tau \to -\tau$ is an isometry, so that one can argue as in [42] that the space can be continued through the coordinate singularity at $\tau = 0$. Then the region in which τ is real or $t^2 > r^2$ of the brane solution is also a complete non-singular solution. The behaviour of the dilaton at $\tau = 0$ will depend on the coefficients β, γ, but in many cases (including the E4-brane, which has constant dilaton), it can be continued smoothly through $T = 0$.

There are also solutions (8.8) with

$$c' + \frac{b^{n-1}}{\sigma^{n-1}}\,, \tag{8.19}$$

where the proper distance is $\sigma^2 = r^2 - t^2$ that are regular for $t^2 < r^2$ with c', b^{n-1} real and non negative. If $n-1$ is a multiple of 4, this a continuation of the solution (8.8) and (8.12) to the region $t^2 < r^2$, but for other n it should be regarded as a distinct solution. The analysis is similar to that of the region $\tau^2 > 0$. In the region $r^2 > t^2$, we use (8.8) with (8.19) and define the coordinates σ, ξ by

$$r = \sigma\cosh\xi\,, \qquad t = \sigma\sinh\xi \tag{8.20}$$

so that the metric becomes

$$ds^2 = \frac{\sigma^\beta}{a^\beta} dx_{||}^2 + \frac{b^2}{\sigma^2} d\sigma^2 + b^2 d\hat{\Omega}_{\bar{N}}^2 \,, \tag{8.21}$$

where

$$d\bar{\Omega}_{\bar{N}}^2 = -d\xi^2 + \cosh^2 \xi d\Omega_N^2 \,. \tag{8.22}$$

If N is a round $n-1$ sphere, then this is the n-dimensional de Sitter metric of "radius" 1, while more generally it is a de Sitter-like cosmology with spatial section N instead of S^{n-1}. The metric

$$ds^2 = \frac{\sigma^\beta}{b^\beta} dx_{||}^2 + \frac{b^2}{\sigma^2} d\sigma^2 \tag{8.23}$$

is the metric on the hyperbolic space H^{m+1}: the change of variables $X = (\beta/2b^{1+\beta/2})$ $\sigma^{\beta/2}$ brings the metric to the standard metric on H^{m+1} $ds^2 = (4/\beta^2) ds_H^2$ where

$$ds_H^2 = \frac{X^2}{b^2} dx_{||}^2 + \frac{b^2}{X^2} dX^2 \tag{8.24}$$

with dilaton

$$e^\phi \propto X^{-2(n-1)\gamma/\beta} \,. \tag{8.25}$$

The boundary of H^{m+1} is the sphere S^m given by the hyperplane $X = 0$ (plus a point at infinity). In the brane solution (8.8), it is at an infinite geodesic distance, with respect to $d\tilde{s}^2$, from any interior point to the boundary $X = 0$ (corresponding to $\sigma = 0$ if $\beta > 0$ and $\sigma = \infty$ if $\beta < 0$), and again the solution is complete, although the dilaton can blow up at the boundary if $\gamma/\beta > 0$.

To summarise, the metric (8.8) has two regions, $\tau^2 > 0$ and $\tau^2 < 0$, and each region can be a complete space. This complete space typically interpolates between a "near-horizon geometry" X and an asymptotic region where $H \sim 1$, which is Minkowski space in $n+m+1$ dimensions $\mathbb{R}^{n+m,1}$ when N is a sphere S^{n-1}, and more generally is the product of $\mathbb{R}^{m,1}$ and a cone over N. If H is of the form (8.12), then the region $\tau^2 \geq 0$ is a complete space whose near-horizon geometry is $X = dS_{m+1} \times H^n$ when N is a sphere S^{n-1}, and more generally is the product of dS_{m+1} and the space $\tilde{N}$ with metric (8.15). If H is of the form (8.19), then the region $\tau^2 < 0$ is a complete space whose near-horizon geometry is $X = H^{m+1} \times dS_n$ when N is a sphere S^{n-1}, and more generally is the product of H^{m+1} and the space $\hat{N}$ with metric (8.22).

9. Holography

In [23], N parallel D3-branes separated by distances of order ρ were considered and the zero-slope limit $\alpha' \to 0$ was taken keeping $r = \rho/\alpha'$ fixed, so that the energy of stretched strings remained finite. This decoupled the bulk and string degrees of freedom leaving a theory on the brane which is $\mathrm{U}(N)$ $\mathcal{N}=4$ super Yang-Mills with Higgs expectation values, which are of order r, corresponding to the brane separations. The D3-brane supergravity solution is of the form (8.1) and (8.2) and has charge $q=a^2 \propto N g_s/\alpha'^2$ where $g_s=g_{\mathrm{YM}}^2$ is the string coupling constant and g_{YM} is the super Yang-Mills coupling constant. Then as $\alpha' \to 0$, q becomes large and the background becomes $AdS_5 \times S^5$. The IIB string theory in the $AdS_5 \times S^5$ background is a good description if the curvature $R \sim 1/a^2$ is not too large, while if a^2 is large, the super Yang-Mills description is reliable. In the 't Hooft limit in which N becomes large while $g_{\mathrm{YM}}^2 N$ is kept fixed, $g_s \sim 1/N$, so that as $N \to \infty$, we get the free string limit $g_s \to 0$, while string loop corrections correspond to $1/N$ corrections in the super Yang-Mills theory. The energy-scale of the super Yang-Mills theory is associated with the radial coordinate r of the AdS space, and going to the boundary $r \to \infty$ in the AdS space coresponds to taking the ultra-violet limit of the super Yang-Mills theory. The super Yang-Mills theory with UV cut-off Λ can in some ways be thought of as being located at a surface $r=r_0$, with the constant r_0 tending to infinity in the limit $\Lambda \to \infty$.

Similar arguments apply here, with the two E4-brane solutions, given by (8.8) and (8.12) or (8.19) with $n=3$, $m=5$, $N=S^3$ and constant ϕ, with spacelike or timelike interpolations corresponding to whether the separation between the E4-branes that is kept fixed is spacelike or timelike. Recall that the scalars of the super Yang-Mills theory are in a vector representation of the $\mathrm{SO}(5,1)$ R-symmetry, where those in the **5** of $\mathrm{SO}(5) \subset \mathrm{SO}(5,1)$ have kinetic terms of the right sign and correspond to brane separations in the 5 spacelike transverse dimensions, while the remaining ($\mathrm{U}(N)$-valued) scalar has kinetic terms of the "wrong" sign and corresponds to time-like separations of the E-branes.

Consider first the case of N parallel E4-branes of the IIB^* string theory separated by distances of order ρ in one of the 5 spacelike transverse dimensions. We take the zero-slope limit $\alpha' \to 0$ keeping $\sigma=\rho/\alpha'$ fixed, so that the energy of stretched strings remains finite. This gives a decoupled theory on the brane consisting of the $\mathrm{U}(N)$ $\mathcal{N}=4$ euclidean super Yang-Mills, with Higgs expectation values of order σ for the scalars corresponding to the spacelike separations. The corresponding supergravity background is the E4-brane with spacelike interpolation and H given by (8.19), arising from the outside of the light-cone with σ real and positive. We again have $q=a^2 \propto N g_s/\alpha'^2$ and $g_s=g_{\mathrm{YM}}^2$, so that for large N, the system can be described by the IIB^* string theory in $dS_5 \times H^5$ if a^2 is large and by the large N euclidean super Yang-Mills theory when a^2 is small. In the 't Hooft limit, string loop corrections again correspond to $1/N$ corrections in the super Yang-Mills theory. The euclidean gauge theory can be thought of as located at the boundary S^4 of H^5.

For N E4-branes of the IIB^* string theory separated by distances of order T in the timelike transverse dimension, we take the zero-slope limit $\alpha' \to 0$ keeping $\tau=T/\alpha'$ fixed. This gives a decoupled theory on the brane consisting of the $\mathrm{U}(N)$

$\mathcal{N} = 4$ euclidean super Yang-Mills, with Higgs expectation values of order τ for the scalars corresponding to the timelike separations. The corresponding supergravity background is the E4-brane with timelike interpolation with H given by (8.12), arising from the inside of the light-cone with τ real. Again for large N, the system can be described by the IIB^* string theory in $dS_5 \times H^5$ if a^2 is large and by the large N euclidean super Yang-Mills theory when a^2 is small. In this case, the euclidean gauge theory can be thought of as being located at the past (or future) Cauchy surface $\tau = -\infty$ ($\tau = \infty$). It was pointed out in [11] that the past and future boundaries of de Sitter space can be identified by identifying points connected by null geodesics, and that this is natural in the context of de Sitter holography, so that the holographic dual is a field theory on one 4-dimensional boundary, rather than two. The identified pair of boundaries will be referred to as "the boundary".

The 4-dimensional euclidean super Yang-Mills theory has scalars taking values in a lorentizan space $\mathbb{R}^{5,1}$. The vacua split into two branches, depending on whether the expectation value of the scalar fields $v^i =< \phi^i >$ is spacelike or timelike. The branch with $v^2 > 0$ corresponds to E4-branes that are spacelike separated in 10-dimensions and arises as a holographic theory on the boundary of H^5, with the scale of the CFT related to a spatial radial coordinate on H^5, while the branch with $v^2 < 0$ corresponds to E4-branes that are timelike separated and arises as a holographic theory on the boundary of dS_5, with the scale of the CFT related to a timelike coordinate on dS_5. If the H^5 is compactified by identifying under a discrete isometry group, the CFT is identified under the corresponding discrete subgroup of the R-symmetry group, and only the branch with $v^2 < 0$ remains, as the dual theory on the boundary of de Sitter space.

Note that in the above, the dimension of the de Sitter space and of the hyperbolic space are the same. Consider cases in which this generalises to d-dimensional euclidean CFTs with conformal group SO$(d, 1)$ and R-symmetry group SO$(n, 1)$, with $n + 1$ scalars taking values in the lorentzian space $\mathbb{R}^{n,1}$. For spacelike scalar expectation values, the dual theory would arise from a theory on $H^d \times dS_n$ as a theory on the boundary of H^d, while for timelike scalar expectation values, the dual theory would be in $dS_d \times H^n$, with the CFT arising on the boundary of the de Sitter space. The theories on the boundaries of dS_n or H^n in these two cases would then be two branches of an n-dimensional conformal field theory with SO$(d, 1)$ R-symmetry, giving an interesting chain of dualities. Explicit supersymmetric examples of this kind were given in [37].

As a further example, consider as in [23] the type-IIB string with a set of N parallel D5-branes wrapping a torus T^4, with M parallel D1-branes lying in the noncompact direction of the D5-branes. The near-horizon geometry is $AdS_3 \times S^3$ and the bulk theory has 16 supersymmetries. The 2-dimensional dual CFT has $(4, 4)$ supersymmetry and SO(4) R-symmetry. This consists of a $D = 2$ super Yang-Mills, obtained from reducing $N = 2, D = 6$ super Yang-Mills on T^4, and a supersymmetric sigma-model whose target space is the M-instanton moduli space in SU(N) Yang-Mills [23]. Now, if time is also compact, then T-dualising the D1-D5 system in the time direction gives an E2-E6-brane system of the type-IIA^* theory wrapped on T^4, with the E2-branes lying in the non-compact directions of the E6-branes. The near horizon geometry is now $dS_3 \times H^3$ and the corresponding 2-dimensional euclidean conformal field theory has R-symmetry SO$(3,1)$ and (4,4) supersymmetry. (For a discussion of (p,q) supersymmetry in 2 euclidean dimensions, see [43].) The 2-dimensional euclidean super Yang-Mills theory with SO$(3,1)$ R-symmetry is that obtained by reducing $N=2, D=6$ super Yang-Mills on a lorentzian torus $T^{3,1}$, while the sigma-model again has the same instanton moduli space as its target space, but the world-sheet is now euclidean.

As for the AdS case, it seems that the holographic duality between a bulk theory in de Sitter space and a euclidean conformal field theory that formally arises in supersymmetric cases applies more generally. Indeed, as was seen in section 8, de Sitter solutions often have associated euclidean brane solutions interpolating between them and flat space or a conical spacetime, and these can form the basis for an argument in the style of [23]. Further evidence for such a de Sitter holography has been discussed in [11, 13].

It is natural to ask whether such a holographic duality can extend to de Sitter solutions of conventional supergravities of the type discussed in section 2. It seems plausible that for these solutions too there should be spacelike brane solutions interpolating between flat space and the de Sitter space solution which could form the basis of a Maldacena-style argument. However, in this case, all supersymmetries are broken and the usual issues arise as to how far one can trust such arguments in the absence of supersymmetry.

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